Charting Trends in the Mandelbrot Set and their significance for Cosmology

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Abstract

The author has, for more than 25 years, been exploring the Mandelbrot Set in unique ways, by highlighting areas of the Complex plane where the magnitude of the iterand follows a trend. Most notably; he discovered the Butterfly figure which appears when one colors in areas where the iterand's magnitude diminishes monotonically over three iterations, and realized it might have significance for theories of Cosmology. This led to an announcement in Amygdala, and conversations with Benoit Mandelbrot, over a quarter century ago. Advances in the subject of Mathematics, since that time, have affirmed the importance of this result to Physics. However; despite repeated efforts, this material has seen little outside attention during the interim, and the territory the author has mapped out in that time remains largely unexplored. This paper is the first in a series to present this line of research to the larger community.

Introduction

The Mandelbrot Set has been called the most complex object in Mathematics, and it is undoubtedly one of the most fascinating and rewarding mathematical objects to explore, but it has much more significance than just a colorful diversion, or a tapestry of wonderful filigrees and spirals to delight the eye and mind. It is, in fact, a catalog or map describing the complex dynamics for a simple quadratic equation, $z = z^2 + c$, over the parameter space of the complex plane. The significance of the Mandelbrot Set to Physics is hard to over-estimate, for reasons that will be spelled out later in this article. Simply stated; the trends found within $\mathcal{M}$, for the case of the Complex numbers, $\mathbb{C}$, also apply in the case of the Quaternions and Octonions, $\mathbb{H}$ and $\mathbb{O}$. The relevance of the complex and hyper-complex number types to Physics has been emphasized in recent work by Michael Atiyah [], and by John Baez and John Huerta [], and it figures prominently into the work of Geoffrey Dixon [], Richard Lockyer [], and Frank D. 'Tony' Smith []. While the Reals are scalars – constant, fixed, or static quantities – the other number types, $\mathbb{C}$, $\mathbb{H}$, and $\mathbb{O}$, display or embody increasing levels of dynamism – because of their imaginary dimensions – and this property becomes apparent under the iteration of functions within their domain.

I was fascinated and mesmerized, when I read about the Mandelbrot Set ($\mathcal{M}$) in Scientific American, in August of 1985 [], and saw for the first time images of this remarkable figure. A.K. Dewdney gave enough information about how a simple set of calculations and program loops could generate the wonderful complexity of $\mathcal{M}$, so I could write a program and create images on our IBM PC. But having friends with better programming skills and tools allowed me to advance the cause more quickly, devising better and quicker ways to get the images to the computer screen. So when I had ideas about how to make the algorithm go quicker; I was visiting a friend, Mark Little, who was able to turn those ideas into code rather swiftly. Looking for a shortcut; I reasoned that since infinite series whose terms diminish over successive values converge, this might work for the Mandelbrot Set. So Mark altered the code to save prior values of the variable 'size' – the iterand magnitude – and then we put in a conditional which tested for the condition: current size < last size < previous size. Our test program called that a point in $\mathcal{M}$, and went on to the next location/pixel, however what we found was not what we expected, as the periphery was full of holes. My immediate reaction was that it looked like a graph of the Cosmological Eras, which I had just learned about in Cosmology class at Dutchess Community College.

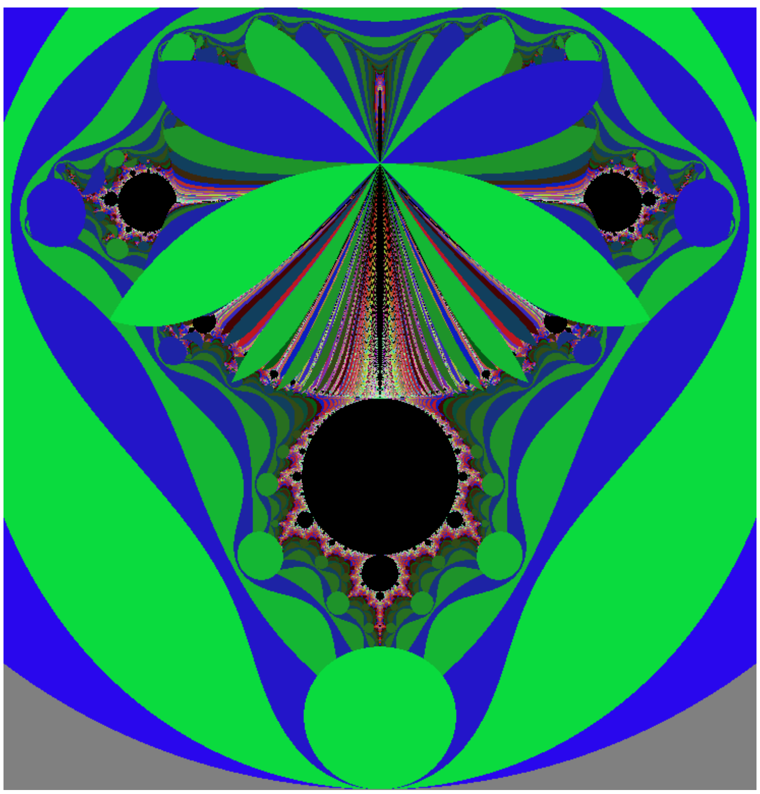

Figure 1 – the Mandelbrot Butterfly

Once we knew our alteration was not a shortcut; we decided to color in those pixels which meet the condition of diminishing iterand magnitude (size), using the color for the number of iterations reached, instead of painting them black. What we ended up with was what I call the Mandelbrot Butterfly – shown in figure 1 above. In this rendering; the green wings and discs depict regions where the iterand comes to be monotonically diminishing on an even-numbered iteration, and the blue ones depict areas that resolve instead on odd-numbered iterations – so it is easy to see the distribution of those sectors within the figure. The Mandelbrot Butterfly is not an entirely separate object, however, but the depiction of a fundamental behavior of the Mandelbrot Set that is seldom observed – because the standard algorithms do not reveal it. So the Mandelbrot Butterfly and the family of related figures catalogues the pre-periodic behavior of the function $z = z^2 + c$ under iteration, revealing something fundamental within the hidden structure of $\mathcal{M}$, while providing a whole new universe of form to observe and explore.

The remainder of this paper will further describe what the author has learned about trends within $\mathcal{M}$, and in the form at is periphery, discussing how this knowledge may have specific relevance to Theoretical Physics and Cosmology, and showing why it must have significant relevance to Physics in general. The use of Julia Sets in Complex Analysis is widespread, and the Mandelbrot Set is a veritable catalog for a whole family of Julias because they possess a form that is self-similar with their point of origin within $\mathcal{M}$, as first noted by Tan Lei []. And as Robert L. Devaney describes []; locations on the periphery of $\mathcal{M}$ can be 'read' from the attributes of the form we find there, as the number of 'spokes' at a branch point tell us the period of the 'bulb' to which it is attached, and so on. Given the above; it is curious that some of the ideas put forth in this paper are not more generally known in the Mathematics community, because their significance is obvious. Until now, however; the author has focused on the possible relevance of this work to Physics, laying a groundwork for future development of physical theories derived from $\mathcal{M}$, making no significant attempt to reach out to the Math or Fractal communities. More recently; a broader and more deliberate outreach has begun, because it is apparent that discoveries made over a quarter century ago have importance for a larger range of fields than was previously imagined.

Constructing $\mathcal{M}$ from First Principles

When attempting to describe what the Mandelbrot Set is; it is simplest to speak of how it is generated or constructed, as this is how the figure is defined. But rather than start with the generating equation for $\mathcal{M}$, I will begin with a brief discussion of the behavior of a simpler equation under indefinite iteration. Considering the iterated squaring function over the positive Reals, $z_n = z_{n-1}^2$ where $z \in \mathbb{R}_+$; we note that $z = 1$ is a critical point where any value of $z_0 < 1$ goes to 0 and any value of $z_0 > 1$ proceeds toward infinity when the squaring function is iterated. Restating this; any $z \leq 1$ remains bounded, while any $z > 1$ will exceed any bound, under iteration. More specifically; a value of exactly 1 remains at the boundary, and any value of z less than 1 will diminish with each iteration of the squaring function – so it remains bounded too – while any value even minutely greater than 1 will increase indefinitely when the function is iterated. This simple example provides some important insights, and is a toy model of the Mandelbrot algorithm. First; iterated squaring, where each successive value is multiplied by itself, is an essential feature of the generating equation for $\mathcal{M}$. Second; the example illustrates that magnitude can be greater or less than a specified value. And third; it shows how for some range of initial values the result remains bounded, while for other values the function goes to infinity, under iteration.

The formula for generating the Mandelbrot Set is only a little more complicated than the example above; instead of $z = z^2$ we have $z = z^2 + c$, and instead of the number line of the positive Reals we have the Complex plane as a domain. Specifically; for our toy model, we have $z_n = z_{n-1}^2$ where $z \in \mathbb{R}_+$, and to create the Mandelbrot Set we use $z_n = z_{n-1}^2 + z_0$ where $z \in \mathbb{C}$. As in our toy model; we want to know which points remain bounded when the function is iterated indefinitely, as these are the points in $\mathcal{M}$. Of course, there is more to constructing a program which creates a map or image of $\mathcal{M}$, but the description above is the entire definition of the Set. If we extend examination for our toy model to include the equation's behavior over the negative Reals, as well as the positive ones, we find that the simple squaring function is symmetric about 0, so we see that the entire range where it remains bounded is $-1 \leq z \leq +1$. Furthermore; the squaring function remains symmetric about 0 in the Complex domain, as well. However; the Mandelbrot function is *not* symmetric about 0 *along* the Real axis, though it *is* symmetric *about* the Real axis – in the direction of the positive and negative imaginaries. That is; $\mathcal{M}$ features a directional progression of form along its periphery, when proceeding from the cusp at (.25, 0i) along either edge, then to the tip at (-2, 0i) – but displays exact mirror symmetry across the Real axis (see fig. 2 below).

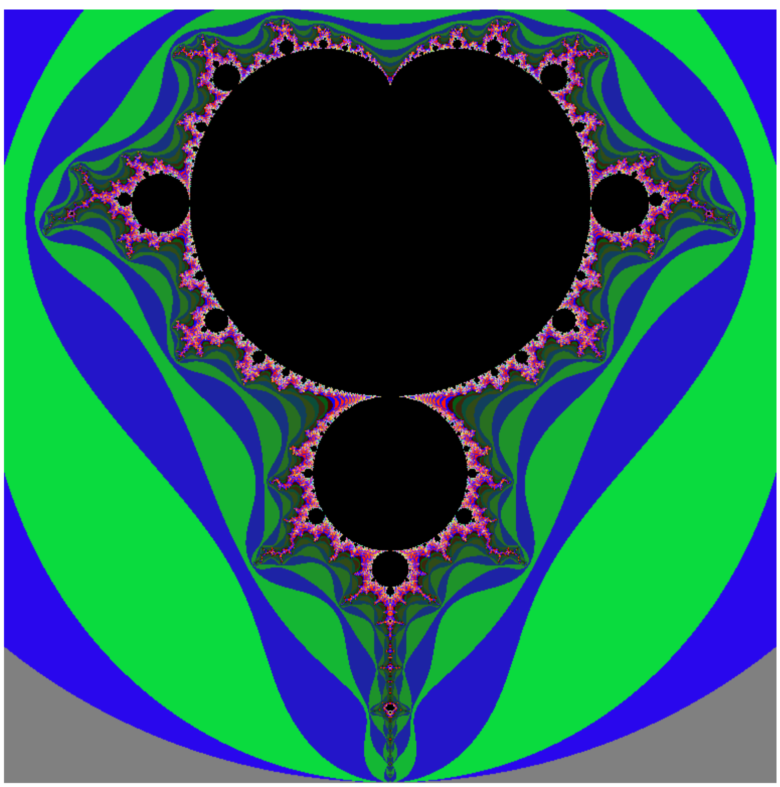

Figure 2 – The Mandelbrot Set (with Real axis vertical)

It is precisely this directional progression and asymmetry along the Real axis that makes the Mandelbrot Set an object of value to Physics, because it illustrates how symmetry is broken in nature. However, the Julia Sets corresponding to any point in the Mandelbrot Set display perfect symmetry about the center point, and they share the same formula – with a variation in how it is applied. Because of their importance to complex dynamics and analysis, I will take a moment to discuss the differences between the Mandelbrot and Julia Set algorithms. It is basically a matter of which parts of the equation vary with the location we are examining on the Complex plane. Instead of the base equation $z = z^2 + c$ becoming $z_n = z_{n-1}^2 + z_0$ as for $\mathcal{M}$; the Julia Sets use $z_n = z_{n-1}^2 + c$, where c is a point in, or near the periphery of $\mathcal{M}$ – a complex number of the form $a + bi$ – and this value is called the seed. Simply put; in the Mandelbrot Set algorithm, the c in $z = z^2 + c$ becomes z_0, the location on the Complex plane for the point we are iterating – and this location is updated for each pixel on the screen, or in the rendered image. In the Julia Set algorithm, the c remains a constant for the entire Set – regardless of the location of the specific point being calculated – and the location, z_0, is used only as the starting point for iteration of the equation. Thus; the Julia Set for a given seed value c is really a projection onto the entire Complex plane, of that seed point or its location relative to structures in $\mathcal{M}$, allowing us to explore its dynamics.

What this means is that we may choose any point on the periphery of the Mandelbrot Set and examine its dynamics apart from the larger structure, as the corresponding Julia Set. The fact that the form along the periphery of $\mathcal{M}$ follows an orderly progression allows us to choose seed points which feature dynamics similar to the kinds of phenomena we wish to study, and that is a major thrust of the author's research, which will be discussed later in this paper. But to complete the picture of how $\mathcal{M}$ can be constructed; we must first entertain the question of how distance may be calculated or estimated, starting with the case for $z \in \mathbb{C}$ the Complex or Argand plane. Complex numbers take the form $a + bi$ and this gives us the option to triangulate distance using the Pythagorean theorem, where the square of the length of two sides of a triangle equals the square of the hypotenuse – which is generally stated as $c^2 = a^2 + b^2$. This works out nicely, because we have two coefficients a and b, so long as we do not imagine that c means the same thing as in the earlier equations. Here c represents the distance from the origin on the Complex plane $(0, 0i)$, where a and b are the Real and imaginary coordinates respectively. That is; where $z = a + bi$ we square both components and sum to find the square of the distance from the origin.

Mathematically speaking; these are all the tools we need, so long as we are talking about $z \in \mathbb{C}$ as with $\mathcal{M}$ and the Julia Sets – the way they are conventionally constructed. There is a broader definition, and as I mentioned; these objects exist not only in $\mathbb{C}$, but also in the domain of $\mathbb{H}$, and $\mathbb{O}$, the Quaternions and Octonions. However; it is not such a simple matter to calculate quaternion and octonion fractals, because their domain is non-commutative and non-associative respectively – so distance estimators replace the exact triangulations of distance possible for the Complex numbers, in some measure. I will leave aside further discussion of hyper-complex fractals for now, and talk more about them in another paper. At this point; I need only remind the reader of my earlier statement that trends within $\mathcal{M}$'s generating procedure, and along its periphery in the Complex domain, are carried over into the patterning of its higher-dimensional cousins. For the 2-dimensional case, the Complex numbers, it is a simple matter to program loops that for each location on a grid of pixels, will calculate a specific complex number iteratively – using the rules outlined above. What is typically done (as in fig. 2 above) is that an iteration limit is set, of perhaps 1000 calculations, and any point that remains bounded is called a point in $\mathcal{M}$, and painted black, while points which exceed a set bound (typically 2), are painted in using a color determined by the number of iterations they attain before going beyond the boundary radius.

The Mandelbrot Mapping Conjecture

The author's observation, when the silhouette of the Butterfly figure was first seen, was that there is an analogy in the forms begat by $\mathcal{M}$ – with the cosmological eras. This was perhaps influenced by the fact that he had just completed coursework in Astrophysics and Cosmology – which focused on the Big Bang model [] and mentioned the recently proposed Inflationary universe scenario (I graduated in 1984 and Guth first published in 1980). So it is understandable that I exclaimed "Oh my – it's the Big Bang" or something similar. I did not, however, fully appreciate the implications of such a claim – not until much later. It should be understood that to prove or disprove such a thing; one must undertake the study of the whole of theoretical Physics, with a special emphasis on Cosmology – because the Mandelbrot Set is seen to describe the entire evolution of the Cosmos and the full range of forces which bring the universe into being. Of course; such a task does not need to be undertaken as a single act, but can be broken up into an array of specific correspondences and testable claims, or related topics of study, which can then be investigated independently. Accordingly; the author has researched and elucidated various elements of this program, and proven the soundness of bits and pieces of his reasoning, while being less specific about the nature of that larger program. However; it became apparent quite recently that what is needed is a concise statement of the research direction that was inspired by my discoveries in the '80s.

Only a few days ago I decided to frame a conjecture, to elucidate this work formally, and I will call it the Mandelbrot Mapping Conjecture. It can be stated thusly. "There exists sufficient dynamical variety in the Mandelbrot Set such that any physical reality or process has a correspondence with a specific location in $\mathcal{M}$." My caveat here is that I am talking about the Physics characterizing a process, or a location in space at a particular time, and not all of the mundane observables and subjective qualia associated with a specific place and time. Studying this is expected to yield insights into the larger theoretical background in which the laws of Physics emerge, and into the means by which the laws of nature came to be what they are now. Perhaps more importantly; since $\mathcal{M}$ describes the evolution of the entire universe over cosmic time, it can provide insights into the stage of evolution that we now inhabit, and its relationship with the larger scope of universal development. Intriguingly; the theoretical framework for Cosmology inspired by this Conjecture supports both a universe with a cold dark end and a cyclical model equally well, depending on the exact details of interpretation, and the way in which $\mathcal{M}$ is depicted (see fig. 3 below). Further; if we admit that patterns and trends within $\mathcal{M}$ in $\mathbb{C}$ extend to the higher-dimensional cases ($\mathbb{H}$ and $\mathbb{O}$), this model allows us to look into a range of Multiverse scenarios as well. In the collection of mini-$\mathcal{M}$s around the periphery, one might see a whole broad collection of universes, but I shall leave that possibility to the reader's imagination for now.

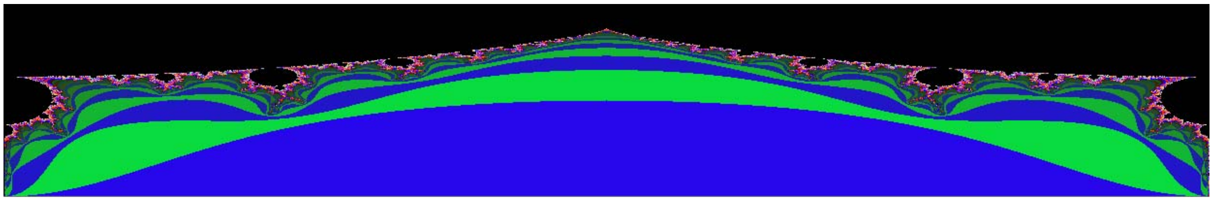

Figure 3 – the Mandelbrot Set in a flat projection (inside is up)

In the rendering of $\mathcal{M}$ above; a range of concentric circles centered at the origin $(0,0i)$ is displayed as rows of pixels – across the screen from left to right – so that the distance of various structures from the origin is depicted as distance from the top of the figure. In this manner; it is easy to see that a ball placed at the top would roll down from the cusp, and depending on its size might fall into one of the open spaces which are the bulbs around the periphery of $\mathcal{M}$. That is; the cusp $(.25,0i)$ is the place with the highest potential energy in this figure, and it spontaneously breaks the symmetry such that the lower potential wells become more and more likely trajectories – over time. However; while a small enough ball is likely to fall into a well further up, a larger ball has no choice but to roll all the way down the hill. Of course; this analogy depends upon the boundary of $\mathcal{M}$ having a solid surface with a gravitational pull toward the bottom of the figure, upon which we might roll a ball, but this may turn out to be an apt comparison. It is seen that the cusp is the point on the boundary where the repeller sets (places almost in $\mathcal{M}$) require the highest number of iterations to calculate, while the point $(-2,0i)$ (in the lower-left and lower-right corners above) is found to be the region of $\mathcal{M}$ where the repeller sets surrounding it need the fewest iterations to resolve. In the depiction above; it is seen that the repeller sets slope away from the cusp, and form wells around each bulb at the periphery of $\mathcal{M}$. But in the view offered in fig. 1 or 2; it is possible to visualize $\mathcal{M}$ instead as a giant thermometer – another useful analogy.

Looking at Trends in the periphery of $\mathcal{M}$

The most obvious trend in the Mandelbrot Set is its asymmetric progression of form, for different Real coordinates. While any Real-numbered value > 0.25 will go to infinity, the negative Reals are bounded at -2 instead. Visually speaking; both the cusp on one end and the spike on the other point toward the negative, along the Real axis. So there is a decided directionality to this figure, *along* that axis, but it is mirror-symmetrical *about* the Real axis, in the direction of the (negative and positive) imaginaries. Features of $\mathcal{M}$ in certain regions of either half display a chirality specific to that region, and opposite to their mirror-image counterparts – which attributes (under the Mandelbrot Mapping Conjecture) observables like the preponderance of left-handed neutrinos to our inhabiting a particular region of space (our local neighborhood) or specific universe, or to a choice of time direction by the early cosmos.

My guess is that whatever direction is forward in time for a given sector of the universe will be perceived as the forward-motion of time by its inhabitants – regardless of how it might appear to a distant observer. I think we are forbidden to see a reverse-time branch of the universe (should it exist), because it either split off before decoupling and is thus obscured from view, or it lives in a time that hasn't happened yet. As in the 'Spontaneous Inflation' theory of Carroll and Chen [], cosmologies based on $\mathcal{M}$ or the MMC allow or feature time that goes both ways, so such questions are germane to their study. But the preference nature displays for a particular chirality or handedness in its forms (a preponderance of matter over antimatter) may thus be explained by the geometry of $\mathcal{M}$.

What I say above will be made clear if the reader understands the earlier statement about $\mathcal{M}$ being like a giant thermometer – describing a range of energies and temperatures from those found in the primordial first fleeting moments of the Cosmos' origin to the depths of its final eon. Of course; the middle range is also clearly delineated. So the reader should rest assured that there is a range of locations within $\mathcal{M}$ in which a condition for sustaining life forms such as our own is possible. But it should be understood that $\mathcal{M}$ also reveals the processes that pertain to a regime where the familiar fundamental forces were unified – or undergo unification – energies and spaces which we may never be able to directly observe. So we might ask what $\mathcal{M}$ tells us about the limit of high-energy Physics near the Planck scale. The cusp at $(.25, 0i)$ is seen as the place corresponding to the first broken symmetry, which in the author's formulation is the direction of time. That is; the flow of time is seen to proceed away from the cusp along either edge of the figure's periphery, but this requires a choice of the left or right hand branch at that point – where time is moving in the opposite direction in the other branch. So the cosmic direction of time seems to be set at the UV limit, during the very first instant of the Big Bang. But there would be an opposite branch, or perhaps an array of universes with a specific time direction – in the higher-dimensional case – as with Carroll and Chen's 'Spontaneous Inflation' theory.

The Mandelbrot Set and its Julias

There is a fundamental similarity between the form found at a given point in the Mandelbrot Set, or along the periphery of $\mathcal{M}$, with the Julia Set associated with that point, or using those coordinates as a seed value. This correspondence was first noted by Tan Lei [] and later by Dierk Schleicher [] and others. It is especially noticeable at the Misiurewicz points, which are infinite repeating points with a pre-period, lying at a branch point or terminus of one of the tendrils or antennae that extend outward from the periphery of the object, or at an 'inflection point' on such a tendril.